A *MATLAB*-Based Transmission-Line Virtual Tool: Finite-Difference Time-Domain Reflectometer

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Abstract

This article introduces a simple virtual tool, *TDRMeter*, for the investigation and visualization of time-domain pulsed voltage/current, traveling along a terminated finite-length transmission line, without and with faults somewhere between the source and the load. The package can be used as an educational tool in various undergraduate lectures to aid in teaching electromagnetics as well as transmission lines.

Keywords: Transmission lines; FDTD methods; pulse measurements; transient propagation; MATLAB; graphical user interfaces; simulation software; circuit simulation; visualization; electrical engineering education; time domain reflectometry

1. Introduction

A set of virtual tools has been introduced to the attention of the *IEEE Antennas and Propagation Magazine* readers [1-4] that can be used both for educational and engineering purposes. If used properly, these virtual tools certainly increase teaching efficiency in classical undergraduate lectures, such as "Electromagnetic Field/Wave Theory," "Antennas and Propagation," "Wireless Communications," "Engineering Electromagnetics," etc.

Another virtual tool, the time-domain reflectometer (TDR), based on the discretization of the time-domain transmission-line (TL) equations in terms of a finite-difference approximation, is introduced in this paper. Originally developed by Fellner-Feldegg [5] in the late sixties, the TDR has been employed in a variety of fields for determining the electrical parameters of materials over a wide frequency bandwidth. An early form of the TDR – dating from the 1930s – with which most people are familiar is radar. Other forms of the TDR are LIDAR, the coaxial TDR, optical-fiber OTDR, and broadband impulse radars.

The TDR is used to locate and identify faults in all types of metallic paired cables. It can locate major or minor cabling problems, including sheath faults, broken conductors, water damage, loose connectors, crimps, cuts, smashed cables, shorted conductors, system components, and a variety of other fault conditions. The basic principle of the TDR is fairly simple. A pulse is generated and sent down the transmission line. If the line is terminated by a load that is equal to the characteristic impedance of the line, no reflected wave or echo returns to the generator or source; otherwise, reflection occurs. By measuring the time difference between the incident pulse and the echo (reflected pulse), it is possible to determine the distance to the fault that produced the echo pulse. Calculations must include the velocity factor of the cable. Also, detailed analysis of the echo signal can reveal additional details of the faults or reflecting objects.

The speed and accuracy of the TDR makes it today's preferred method of cable-fault location. Although today's instruments are more user friendly, a good understanding of the basic principles and applications of a TDR is essential to successful troubleshooting. The pulse generated by the TDR takes a certain amount of time, and thus distance, to launch. This distance is known as the *blind spot*. The length of the blind spot varies with the pulse width: the longer the pulse width, the larger the blind spot. It is more difficult to locate a fault contained within the blind spot. If a fault is suspected within the first section of the cable, it is advisable to add a length of cable (a jumper) between the TDR and the cable being tested. Any faults that may have been hidden in the blind spot can now easily be located (however, remember to take the length of the jumper cable into account). It is best if the jumper cable has the same characteristic impedance as the cable under test.

2. Time-Domain Transmission-Line Equations

A transmission line (TL) is more than a set of long, parallel lines: it is a distributed-parameter physical system. Transmission lines are used to transmit electric energy and communication signals from one point to another. A basic transmission line connects a source to a load. This may be a transmitter and an antenna, a television or a radio antenna and a receiver, one port of a coupler and a power meter, etc. A typical two-wire transmission line is sketched in Figure 1a. A transmission line has two sets of constitu-

Figure 1a. A finite-length transmission line, the time-domain voltage source, $v_s(t)$, and the source resistor, R_s, terminated by a complex load, R_L.

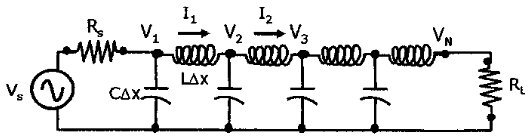

Figure 1b. The loss-free equivalent circuit of the transmission line of Figure 1a. L [H/m] and C [F/m] are the unit-length inductance and conductance, respectively.

tive parameters. The primary parameters are unit-length resistance, R [Ω/m], inductance, L [H/m], capacitance, C [F/m], and admittance, G [S/m]. A circuit equivalent of a transmission line is pictured in Figure 1b. The secondary parameters are the characteristic impedance, Z_0, and the complex propagation constant. The real and imaginary parts of the propagation constant are the loss and phase variation per unit length, respectively.

For a uniform transmission line (along the x direction), the coupled time-domain transmission-line equations of the voltage and current, in differential form, may be written as

$$\frac{\partial v(x,t)}{\partial x}+L\frac{\partial i(x,t)}{\partial t}+Ri(x,t)=0, \tag{1a}$$

$$\frac{\partial i(x,t)}{\partial x}+C\frac{\partial v(x,t)}{\partial t}+Gv(x,t)=0, \tag{1b}$$

where here $v(x,t)$ and $i(x,t)$ are the space- and time-dependent voltage and current, respectively.

2.1 FDTD Representations

The well-known FDTD representations of Equation (1) can be given as [6]

$$v^n(k)=\left[\frac{\frac{C}{\Delta t}-\frac{G}{2}}{\frac{C}{\Delta t}+\frac{G}{2}}\right]v^{n-1}(k)-\left[\frac{1}{\frac{C}{\Delta t}+\frac{G}{2}}\right]\frac{\left[i^n(k)-i^n(k-1)\right]}{\Delta x}, \tag{2a}$$

$$i^n(k)=\left[\frac{\frac{L}{\Delta t}-\frac{R}{2}}{\frac{L}{\Delta t}+\frac{R}{2}}\right]i^{n-1}(k)-\left[\frac{1}{\frac{L}{\Delta t}+\frac{R}{2}}\right]\frac{\left(v^n(k)-v^n(k-1)\right)}{\Delta x}, \tag{2b}$$

which connect the nodes of the voltages to the currents in between them. Here, the integers k and n respectively represent spatial (x) and time (t) indices, so that physical space and time values are specified via

$$v(x,t)=v(k,n)=v^n(k),$$
$$i(x,t)=i(k,n)=i^n(k), \tag{3}$$

(by using $x=k\times\Delta x$ and $t=n\times\Delta t$). The leap-frog scheme of Equation (2) is given in Figure 3. As in one-dimensional FDTD simulations, the stability and the discretization conditions are given by

$$\Delta t\le \Delta x/c,$$
$$c=\frac{1}{\sqrt{LC}}, \tag{4}$$
$$\Delta x\le \lambda_{min}/10.$$

2.3 FDTD Representations: Fault modeling Along a Transmission Line

Inhomogeneity of a transmission line can be introduced in various different ways: a series of different transmission lines may be assumed to be cascaded [6], or any or all of the primary

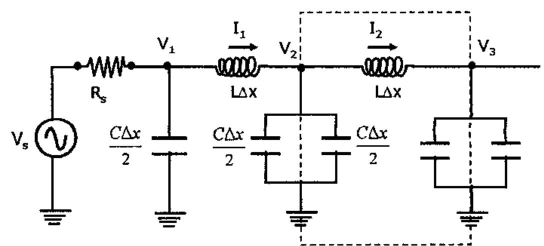

Figure 2. A lossless but symmetrical equivalent circuit in terms of the primary line parameters (V_1, V_2,...,V_N are the line voltages).

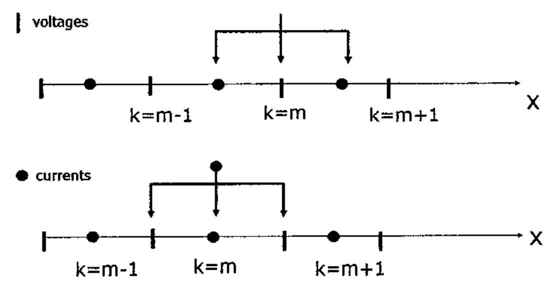

Figure 3. A one-dimensional FDTD leap-frog scheme: voltage and current nodes.

parameters of the line may be assumed to be different at specific nodes. The first approach requires the satisfaction of the continuity equations at the interface, since the iterative equations based on the differential operators become invalid. If the second approach is used, then one needs to replace the primary parameters R, L, C, and G in Equation (2) with R', L', C', and G', where $R' = R + R_f$, $G' = G + G_f$, $L' = L + L_f$, and $C' = C + C_f$, and where R_f, G_f, L_f, and C_f are the unit-length parameters of the fault.

2.4 FDTD Representations: Terminated Lines

The transmission-line representation given in Figure 1a includes a voltage source, $v_s(t)$, with internal resistance R_s at the left end, and a load resistor, R_L, at the right end. Its symmetrical circuit model is given in Figure 2. In this case, Equations (2) are used for all the nodes except the first and last node. For the first node, a Thevenin source, as shown in Figure 1a, can be replaced by its Norton equivalent, as given in Figure 4a, and Kirchoff's current law (KCL):

$$\frac{v_s^n}{R_s} = i^n(1) + C\frac{\Delta x}{2}\frac{\partial v^n(1)}{\partial t} + \frac{v^n(1)}{R_s} \tag{5}$$

can be used to obtain the discrete

$$v^n(1) = \left[\frac{R_s\frac{C\Delta x}{2\Delta t} - \frac{1}{2}}{R_s\frac{C\Delta x}{2\Delta t} + \frac{1}{2}}\right] v^{n-1}(1) - \left[\frac{1}{R_s\frac{C\Delta x}{2\Delta t} + \frac{1}{2}}\right]\{R_s\} i^n(1) - \frac{\left(v_s^n + v_s^{n-1}\right)}{2} \tag{6a}$$

form that can be used for the first ($k = 1$) node. If a similar procedure is followed for the last ($k = N$) node (see Figure 4b),

$$v^n(N) = \left[\frac{R_L\frac{C\Delta x}{2\Delta t} - \frac{1}{2}}{R_L\frac{C\Delta x}{2\Delta t} + \frac{1}{2}}\right] v^{n-1}(N) + \left[\frac{1}{R_L\frac{C\Delta x}{2\Delta t} + \frac{1}{2}}\right] R_L i^n(N-1) \tag{6b}$$

is obtained, and can be used to represent a resistive load termination.

2.5 FDTD Representations: Complex Termination

In practice, transmission lines are mostly terminated by complex loads. For example, for the parallel RC termination (e.g., see Figure 4b) if Kirchoff's current law for the last node,

$$i^n(N-1) = v^n(N)\left(\frac{G\Delta z}{2} + \frac{1}{R_L}\right) + \frac{C\Delta z}{2}\frac{\partial v^n(N)}{\partial t} + C_L\frac{\partial v^n(N)}{\partial t}, \tag{7a}$$

$$v^n(N) = \left[\frac{\frac{C\Delta x + 2C_L}{2\Delta t} - \frac{1}{2}\left(\frac{G\Delta x}{2} + \frac{1}{R_L}\right)}{\frac{C\Delta x + 2C_L}{2\Delta t} + \frac{1}{2}\left(\frac{G\Delta x}{2} + \frac{1}{R_L}\right)}\right] v^{n-1}(N) + \left[\frac{1}{\frac{C\Delta x + 2C_L}{2} + \frac{1}{2}\left(\frac{G\Delta x}{2\Delta t} + \frac{1}{R_L}\right)}\right] i^n(N-1). \tag{7b}$$

It should be noted that choosing $R_L = 0\,\Omega$ for the parallel RC load must reduce to a short-circuit termination; unfortunately, Equation (7b) does *not* automatically yield this result. Therefore, Equation (7b) must be replaced with $v^n(N) = -v^{n-1}(N)$ if $R_L = 0\,\Omega$ is selected for the parallel RC termination. The iterative equations for all the other complex terminations may be obtained by following similar procedures [6].

2.6 Source Specifications

Numerical dispersion determines the selection of the discretization step, Δx (Δt is determined by the Courant stability criterion) [8]. In practice, discretization values between $\Delta x = \lambda_{min}/10$ and $\Delta x = \lambda_{min}/150$ are appropriate, depending on whether or not the phase information is required. When pulsed signals are used, the highest frequency content of the pulse should be taken into

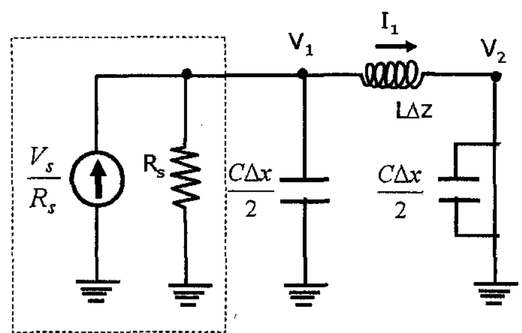

Figure 4a. The source injection at node 1, after the Thevenin-to-Norton transformation.

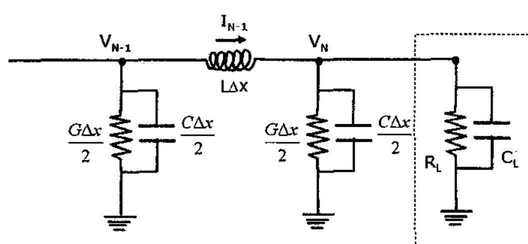

Figure 4b. The load model at the last node (the line is terminated by a parallel *RC* pair).

account. The ideal rectangular pulse has zero rise and fall times, which means it has an infinite number of sinusoids in the frequency domain. Therefore, numerical dispersion effects are unavoidable for the rectangular-pulse source. On the other hand, the discretization parameters can be specified accordingly for Gaussian and/or trapezoidal pulses. For example, the duration of a Gaussian pulse may be specified automatically in terms of Δx. For this purpose, Δx is first specified in terms of the length of the transmission line and the number of nodes. Then, the time step, Δt, is calculated from Equation (5a). The bandwidth of the Gaussian pulse may be specified as $B = f_{max} = c/(10\Delta x)$ if the spatial extent of the pulse is assumed to be $10\Delta x$. Finally, the two parameters of the Gaussian pulse $g(t) = \exp\left[-\alpha(1-\varphi)^2\right]$ may be chosen accordingly to be $\alpha = 3.3B^2$ and $\varphi = 4/\sqrt{\alpha}$. In this case, $\lambda_{min}/10$ sampling is automatically guaranteed [8].

3. The *MATLAB*-based *TDRMeter* Package

The *TDRMeter* package is a multipurpose program designed in *MATLAB 6.5* (the reader is strongly advised to review transmission-line theory, in both the time and frequency domains, if he or she feels uncomfortable with the equations given here. There are excellent classical books and Internet sources, and there is no need to cite them. The authors used [6] and [8], and references listed there, for this purpose.) *TDRMeter* and a set of examples can be downloaded from

http://www3.dogus.edu.tr/lsevgi (or /culuisik).

The front panel of *TDRMeter* is given in Figure 5. There are four input data blocks on top of the panel. The user supplies the unit-length transmission-line parameters on the left. The mid-left block is reserved for the source parameters. The user may select one of three different source types: a Gaussian pulse, a rectangular pulse, or a trapezoidal pulse, along with the pulse duration and rise/fall times (if the source is trapezoidal). An internal source resistor is also supplied inside this block. The pulse length of the Gaussian voltage source is automatically selected, as explained in the previous section. The third block on the mid-right is used for the specification of the load. It may be a resistive load, a parallel combination of a resistor and a capacitor, or a serial combination of a resistor and an inductor, and serial/parallel resonant circuits. The user selects the load type from the popup menu and supplies the numerical values. The last block at the right is used for the fault definition. The user is allowed to change only the unit-length parallel elements (*C* and *G*), but this can be easily altered by the user by changing the codes in the *M* file. The transmission-line length and the observation point are supplied at the right top of the front panel. All of the units should be given as specified on the front panel.

Two plots are used for visualization purposes. The outcome of the FDTD simulations can be given as either signal as a function of transmission-line length at each simulation time, or signal as a function of time at the specified observation point. The figure on the front panel is reserved for the visualization of pulse propagation/reflections along the transmission line as time progresses. The user should start time simulations by pressing the *Run* button. The second plot may be recalled by using the "*Plot Sig. vs. Time*" command button.

The popup menu at the right top contains a key selection: the transmission line or the TD (time-domain) reflectometer. The

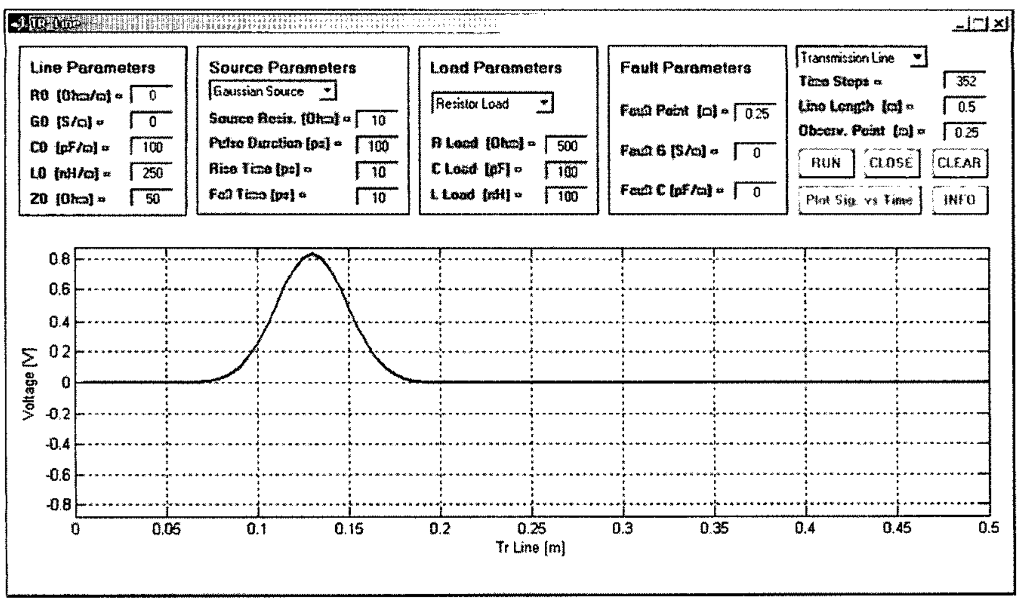

Figure 5. The front panel of the *TDRMeter* package, with a Gaussian pulse traveling towards the load (for a 50 Ω transmission line, R_s = 10 Ω, resistive load = 500 Ω).

default selection is the transmission line, where the user specifies all input parameters, and with which he or she does simulations to visualize time-domain transmission-line effects. The TD Reflectometer selection may be used to locate the transmission-line termination and faults. It is designed in such a way that the user specifies only the line and source parameters; the load is selected automatically (also randomly), and a fault is introduced at an arbitrary point along the transmission line, so the user can find its type and/or numerical values (if possible) by only observing/analyzing the output plots, e.g., signal as a function of time at the selected point on the transmission line.

The plot in Figure 5 belonged to a $50\,\Omega$, 0.5 m, loss-free homogeneous (uniform) transmission line, excited with a Gaussian-pulse source having a $10\,\Omega$ internal resistor, and terminated by a resistive load of $100\,\Omega$ (the transmission line was fault-free, i.e., uniform). The simulation time step was 400. Since the unit length inductance and capacitor values were 250 nH/m, and 100 pF/m respectively, the speed of the voltage and current waves along the line was calculated, from the limiting case of Equation (6a), to be $v = 2$ m/s. The transmission line was divided into 100 nodes; therefore, $\Delta x = 0.5/100 = 5\,\text{mm}$. The time step, Δt, was calculated to be $\Delta t = \Delta x / v = 25\,\text{psec}$. With this choice, the voltage (or current) pulse propagated one node at a time; therefore, it took $100\Delta t$ for the injected pulse to reach the load. The total of $400\Delta t$ resulted in two reflections from the load and two reflections from the ends of the source.

5. Concluding Remarks

A simple transmission-line *MATLAB* package was introduced in this article. The package is design to visualize voltage pulse propagation and reflections from various discontinuities along a transmission line. It may also be used as a TDR. The user may exercise tests of different types of randomly selected terminations and/or faults, and may try to find out what kind of problem exists by analyzing the time history of the simulated voltage pulses. The package is a good virtual tool that can be used in both EM lectures as well as in virtual EM labs.

The package can be easily modified to predict transmission-line parameters, such as the characteristic impedance and the complex propagation constant from the analysis of time-dependent traveling/reflecting pulses by using a known termination, which is left to the reader.

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During the 2006 Joint IEEE International Symposium on Antennas and Propagation, National Radio Science Meeting, and AMERM Meeting, to be held July 9-14, 2006, in Albuquerque, New Mexico, USA, the IEEE Antennas and Propagation Society will sponsor a Sea of Opportunities Pavilion. The purpose of the pavilion is to enhance corporate/academic collaboration in serving the career-development opportunities for the next generation of electromagnetic engineering leaders. University faculty are invited to present a synopsis of emerging students (graduation six to 12 months from the time of the meeting) to potential employers, and companies are invited to present a concise general overview of their company to students. Nominally, each presentation will be ten minutes, although longer presentations may be possible, depending on demand. In addition, AP-S Student Members are encouraged to submit their resumes for a resume book, which will be compiled for the benefit of companies and for students seeking post-doctoral or graduate-student appointments, for use by university faculty. Companies are invited to submit contact information to be distributed to AP-S faculty for students graduating throughout the year. Please indicate if you have or are likely to have positions available for interns, undergrads, MS, and/or PhD graduates.)

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